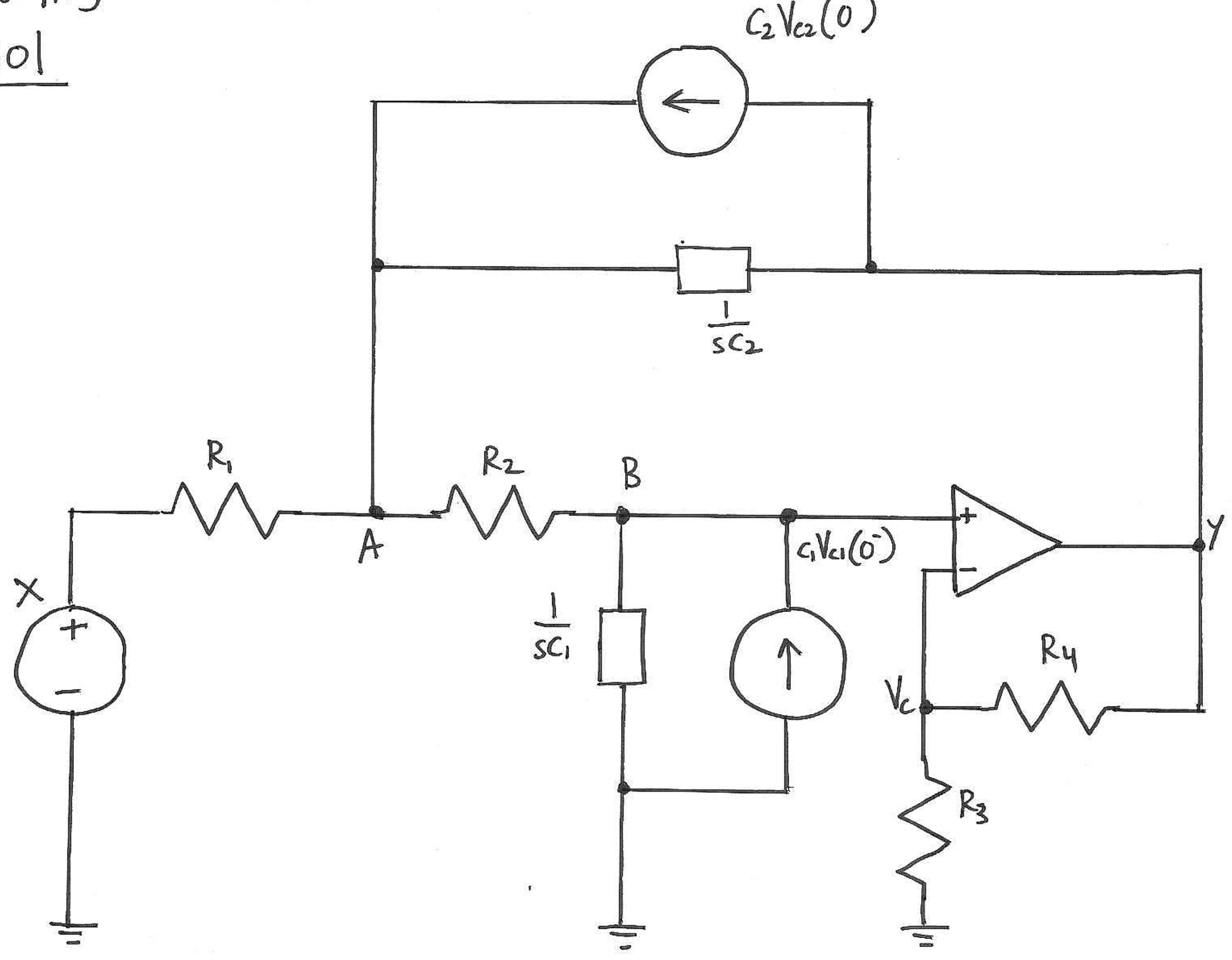
SIGNALS AND SYSTEMS - WEEK 9

Problem 1

Laplace transform frequency dependent components and time-varying sources. For lowpass filter.

sol



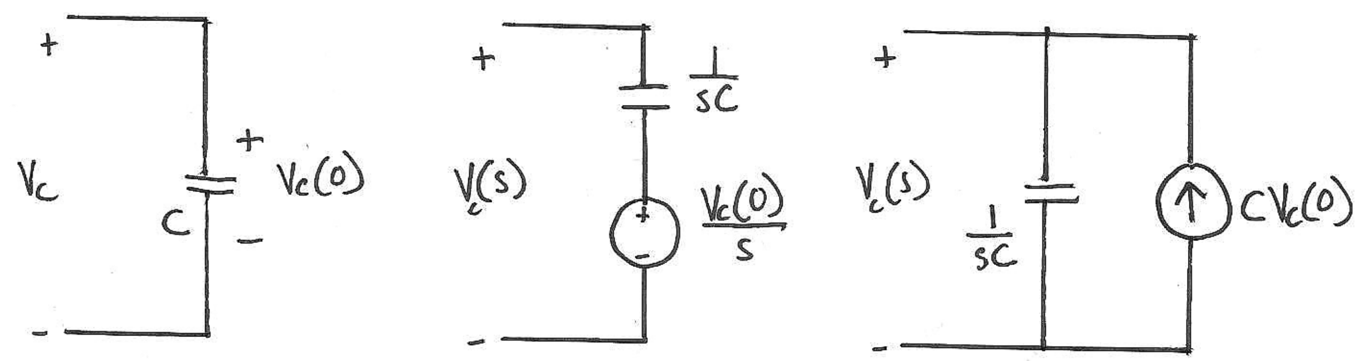
Node equations:

$$\frac{V_A - X}{R_1} + \frac{V_A - V_B}{R_2} + (V_A - Y)sC_2 - C_2V_{c2}(0^-) = 0$$

$$\frac{V_B - V_A}{R_2} + V_B \cdot sC_1 - C_1V_{c1}(0^-) = 0$$

Constraint: $V_B = V_C = \frac{Y}{K}$, $K = 1 + \frac{R_4}{R_3}$

Laplace transform of capacitor with initial voltage.



Note, current source direction is opposite of polarity.

Problem 2

Derive the system response $Y(s) = \frac{P(s)}{Q(s)} X(s) + \frac{I(s)}{Q(s)}$

Sol.

Define the equations in Maple:

$$sys := \left\{ \frac{VA - X}{R_1} + \frac{VA - VB}{R_2} + sC_2 \cdot (VA - Y) - C_2 V_{c2}(0^-) = 0,\ \frac{VB - VA}{R_2} + sC_1 \cdot VB - C_1 V_{c1}(0^-) = 0 \right\}$$

$$VB := \frac{Y}{k}$$

Solve(sys, {VA, Y})

$$Y = \frac{k\left(C_1C_2R_1R_2V_{c1}(0^-)s + C_1R_1V_{c1}(0^-) + C_1R_2V_{c1}(0^-) + C_2R_1V_{c2}(0^-) + X\right)}{C_1C_2R_1R_2s^2 + s(C_1R_1 + C_1R_2 + C_2R_1 - C_2R_1k) + 1}$$

Using Maple's collect (%, X) on the fraction yields:

$$Y(s) = \underbrace{\frac{k}{C_1C_2R_1R_2s^2 + s(C_1R_1 + C_1R_2 + C_2R_1 - C_2R_1k) + 1} X(s)}_{\text{zero state response}} + \underbrace{\frac{k(C_1C_2R_1R_2V_{c1}(0^-)s + C_1R_1V_{c1}(0^-) + C_1R_2V_{c1}(0^-) + C_2R_1V_{c2}(0^-))}{C_1C_2R_1R_2s^2 + s(C_1R_1 + C_1R_2 + C_2R_1 - C_2R_1k) + 1}}_{\text{zero input response}}$$

$$Y(s) = \frac{b_0}{s^2 + a_1s + a_0} X(s) + \frac{\frac{k}{C_1R_2} V_{c2}(0^-) + k\left(s + \frac{1}{C_2R_2} + \frac{1}{C_2R_1}\right)V_{c1}(0^-)}{s^2 + a_1s + a_0}$$

Problem 3

Solve for the zero-input response using Maple.

Sol

$$Y_{zi}(s) = \frac{\frac{k}{C_1R_2} V_{c2}(0^-) + k\left(s + \frac{1}{C_2R_2} + \frac{1}{C_2R_1}\right) V_{c1}(0^-)}{s^2 + a_1 s + a_0}$$

$$Y_{zi}(s) = \frac{2.116}{s+147.306} - \frac{0.116}{s+2680.2}$$

Inverse Laplace transform to find $Y_{zi}(t)$.

$$\mathcal{L}\left\{\frac{2.116}{s+147.3} + -\frac{0.116}{s+2680.2}\right\}^{-1} = 2.11e^{-147.3t} - 0.116e^{-2680.2t}$$

↑

invlaplace

Problem 4

Solve for the zero-state response, to the input $2u(t) = x(t)$.

Sol

$$Y_{zs}(s) = \frac{b_0}{s^2 + a_1 s + a_0} X = \underbrace{\left[\frac{4}{s} - \frac{4.232}{s+147.3} + \frac{0.2326}{s+2680}\right]}_{H(s)} \cdot \underbrace{\frac{2}{s}}_{X(s)}$$

$$\text{invlaplace}\left(\left[\frac{4}{s} - \frac{4.232}{s+147.3} + \frac{0.2326}{s+2680}\right] \cdot \frac{2}{s}, s, t\right)$$

$$Y_{zs}(t) = 4 - 4.232e^{-147.3t} + 0.2326e^{-2680t}$$