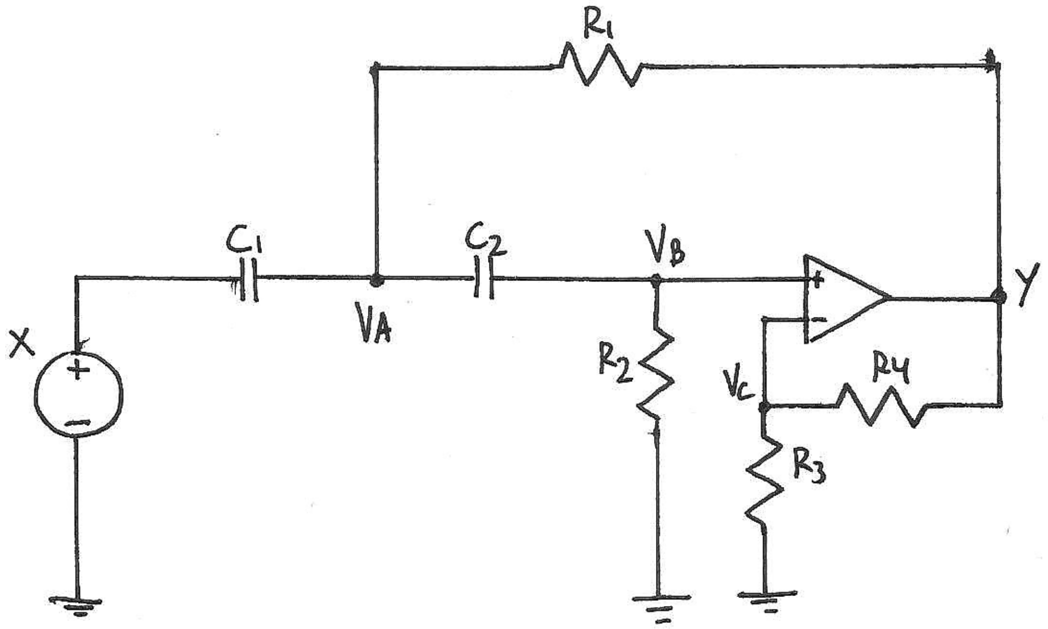
SIGNALS AND SYSTEMS - WEEK 6



Problem 1

Derive the frequency characteristic $H(\omega)$.

Sol

Replace capacitors with their impedance $Z_C = \frac{1}{j\omega C}$, because then we can use ohm's law $I = \frac{V}{Z}$.

$$\frac{V_A - X}{Z_{C1}} + \frac{V_A - V_B}{Z_{C2}} + \frac{V_A - Y}{R_1} = 0$$

$$\frac{V_B - V_A}{Z_{C2}} + \frac{V_B}{R_2} = 0$$

Insert $Z_C = \frac{1}{j\omega C}$.

$$(V_A - X) j\omega C_1 + (V_A - V_B) j\omega C_2 + \frac{V_A - Y}{R_1} = 0$$

$$(V_B - V_A) j\omega C_2 + \frac{V_B}{R_2} = 0$$

Constraint: $V_B = V_c = \frac{Y}{K}$, $K = 1 + \frac{R_4}{R_3}$.

Solving in Maple yields: $H(\omega) = \frac{Y}{X} = \dfrac{K \cdot (j\omega)^2}{(j\omega)^2 + j\omega\left(\frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} + \frac{1-K}{R_1 C_1}\right) + \frac{1}{R_1 R_2 C_1 C_2}}$

Problem 2

Derive the differential equation.

Sol

$$H(\omega) = \frac{k\cdot (j\omega)^2}{(j\omega)^2 + j\omega\left(\frac{1}{R_2C_1} + \frac{1}{R_2C_2} + \frac{1-k}{R_1C_1}\right) + \frac{1}{R_1R_2C_1C_2}} = \frac{Y}{X}$$

$$Y(\omega)\cdot\left[(j\omega)^2 + j\omega\left(\frac{1}{R_2C_1} + \frac{1}{R_2C_2} + \frac{1-k}{R_1C_1}\right) + \frac{1}{R_1R_2C_1C_2}\right] = X(\omega)k(j\omega)^2$$

This equation is in the frequency domain.
To obtain the differential equation, transform back to time.

Time $\leftrightarrow$ Frequency

$\frac{d}{dt} \leftrightarrow j\omega$

$$y(t)\left[D^2 + D\left(\frac{1}{R_2C_1} + \frac{1}{R_2C_2} + \frac{1-k}{R_1C_1}\right) + \frac{1}{R_1R_2C_1C_2}\right] = x(t)\,D^2\cdot k$$

$$\ddot{y}(t) + \dot{y}(t)\left(\frac{1}{R_2C_1} + \frac{1}{R_2C_2} + \frac{1-k}{R_1C_1}\right) + \frac{y(t)}{R_1R_2C_1C_2} = k\ddot{x}(t)$$

Conclusion:
It is easy to find the differential equation via the transfer function, $H(\omega)$.

Problem 3

Determine passband gain and stopband asymptote slope.

Sol

The numerical transfer function is obtained by inserting Component values:

$$H(\omega) = \frac{2(j\omega)^2}{(j\omega)^2 + 0.6283\,j\omega + 0.0987}$$

It's a highpass filter, so passband lies in the high frequencies

Passband gain: $|H(10^{12})| = 2 = 6\ \text{dB}$ $(K = 2)$

The stopband lies in the low frequencies:

Stopband slope:

$$20\log_{10}(|H(10^{-5})|) - 20\log_{10}(|H(10^{-6})|)$$

$$= 40\ \frac{\text{dB}}{\text{dec}}$$

Because it's a $n=2$ order highpass filter, the slope is

$$n \cdot 20\ \frac{\text{dB}}{\text{dec}} = 40\ \frac{\text{dB}}{\text{dec}}.$$

Problem 4

Find phase angle at low and high frequencies.

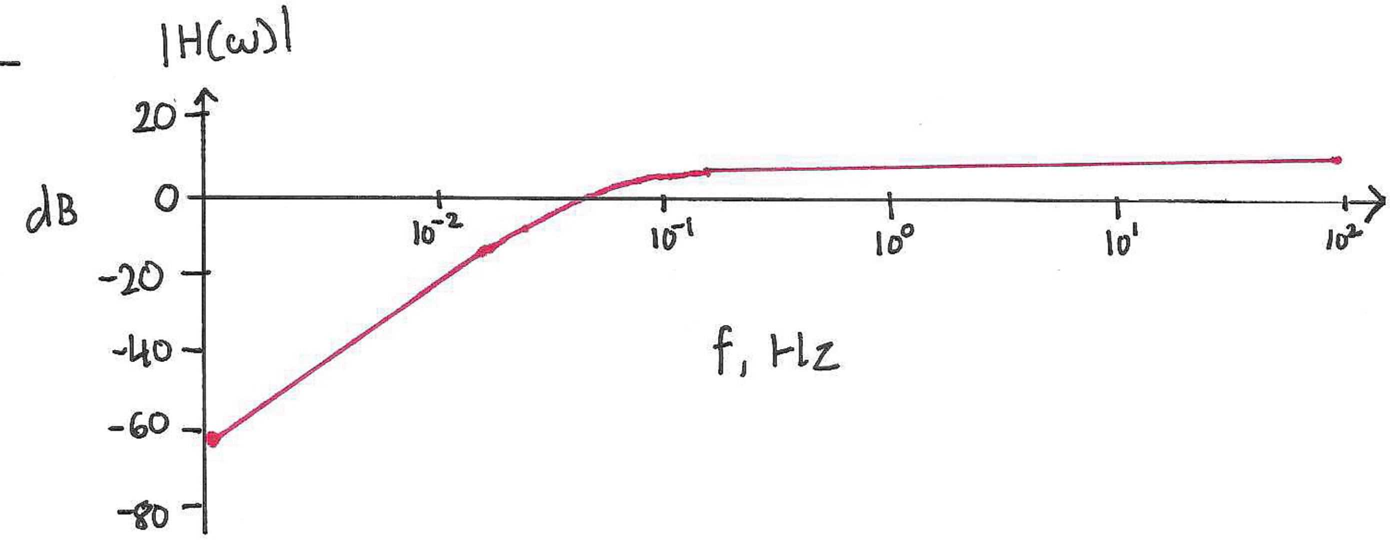
Sol

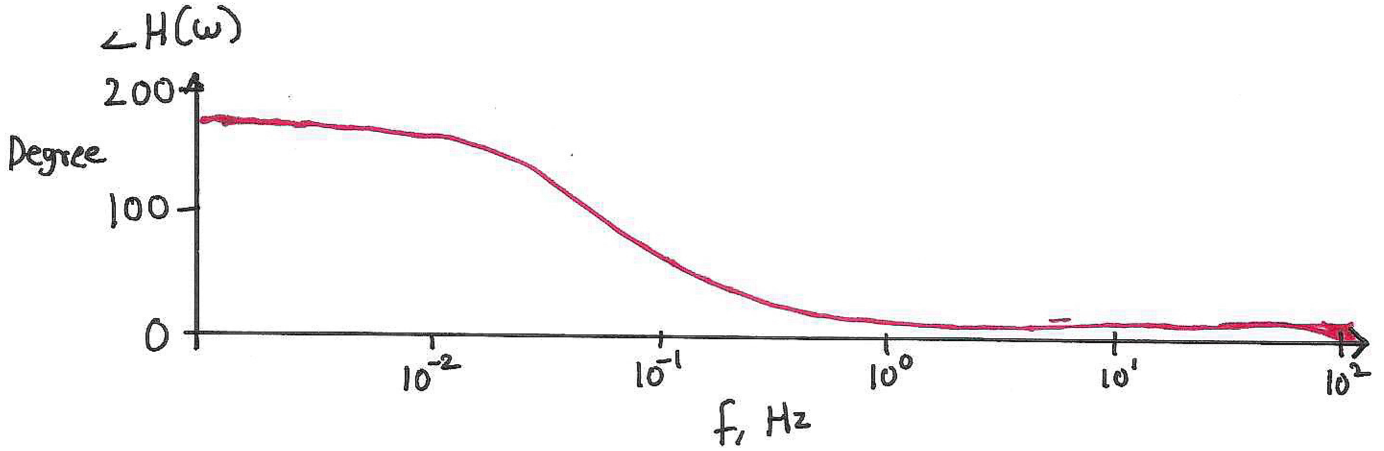
- $\text{Argument}(H(0)) = 180°$
- $\text{Argument}(H(10^{12})) = 0$

Problem 5

Plot the amplitude and phase characteristic.

Sol





Low frequency

$|H(\omega)| = -\infty$

$\angle H(\omega) = 180^\circ$

High frequency

$|H(\omega)| = k$

$\angle H(\omega) = 0^\circ$

For 2nd order Sallen-key highpass filter.