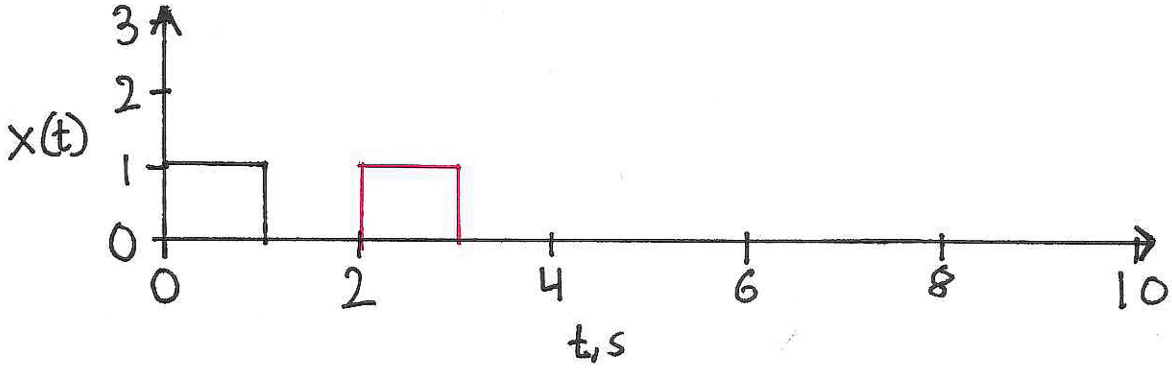
SIGNALS AND SYSTEMS - WEEK 4

Problem 1

Calculate the correlation between the two square waved shaped signals.



sol

The signals can be written using step functions.

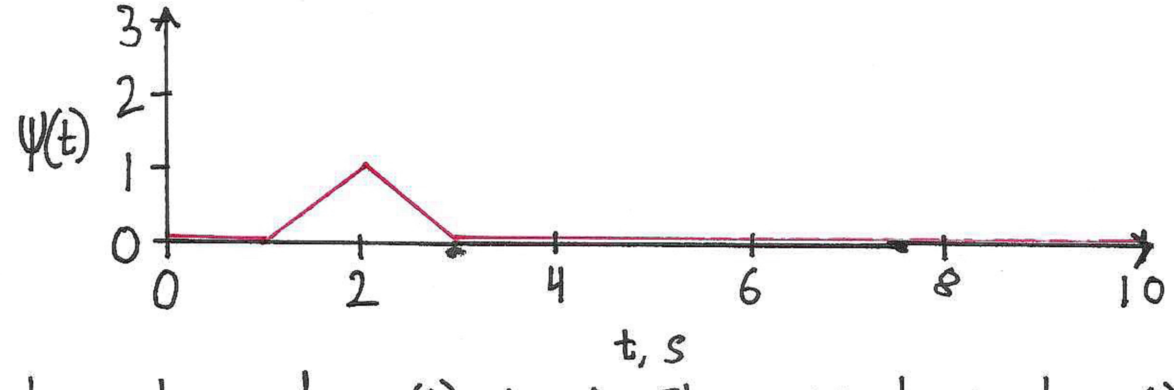
$x_1(t) = u(t) - u(t-1)$

$x_2(t) = u(t-2) - u(t-3)$

$\Big\}$ $u(t)$ = Heaviside(t) in Maple

Correlation integral: $\psi(t) = \int_{-\infty}^{\infty} x_1(\tau - t)\, x_2(\tau)\, d\tau$

This yields a triangular shaped wave:

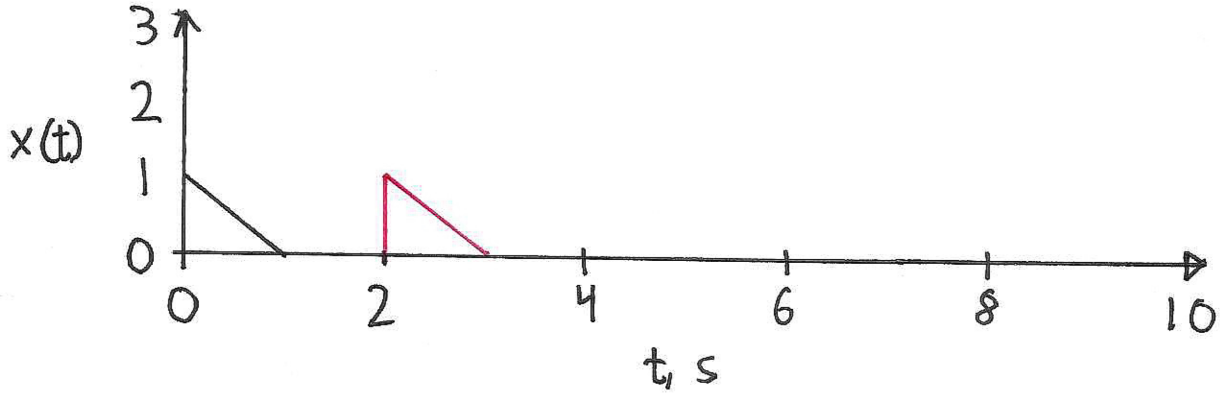


The peak value of $\psi(t)$ is 1. The original signal, $x_1(t)$, has area 1.

This strongly suggests that $x_1(t)$ is contained in $x_2(t)$.

Problem 2

Calculate the correlation of the two triangle shaped signals.



Sol

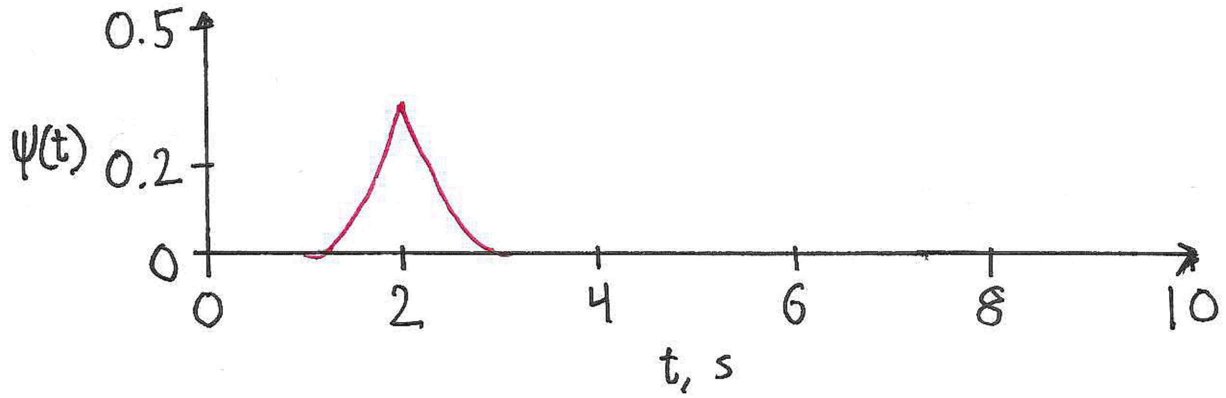
Again, the signals can be defined using straight lines and step functions.

$x_1(t) = (1-t) \cdot u(t) \cdot u(-t+1)$

$x_2(t) = (3-t) \cdot u(t-2) \cdot u(-t+3)$

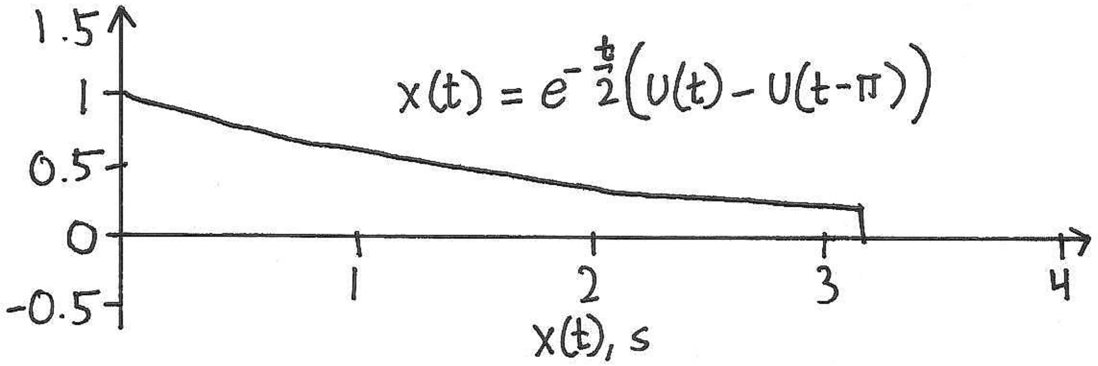
Correlation integral: $\psi(t) = \int_{-\infty}^{\infty} x_1(\tau - t) x_2(\tau)\, d\tau$

This yields a smooth triangular waveform:



Problem 3

Calculate the Fourier coefficients and plot the magnitude and phase.



Sol

Fourier Series only exist for periodic signals.

We assume that the truncated signal x(t) has period $T = \pi$.

$$a_n = \frac{2}{T} \cdot \int_0^T x(t) \cos(n\omega_0 t)\, dt$$

$$b_n = \frac{2}{T} \cdot \int_0^T x(t) \sin(n\omega_0 t)\, dt$$

$$a_0 = \frac{1}{T} \int_0^T x(t)\, dt \qquad \text{(mean value)}$$

Compact form: $C_n = \sqrt{a_n^2 + b_n^2}$, $C_0 = a_0$, $\theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right)$

$$a_0 = \int_0^T x(t)\, dt = 0.504 \quad \Rightarrow C_0 = 0.504$$

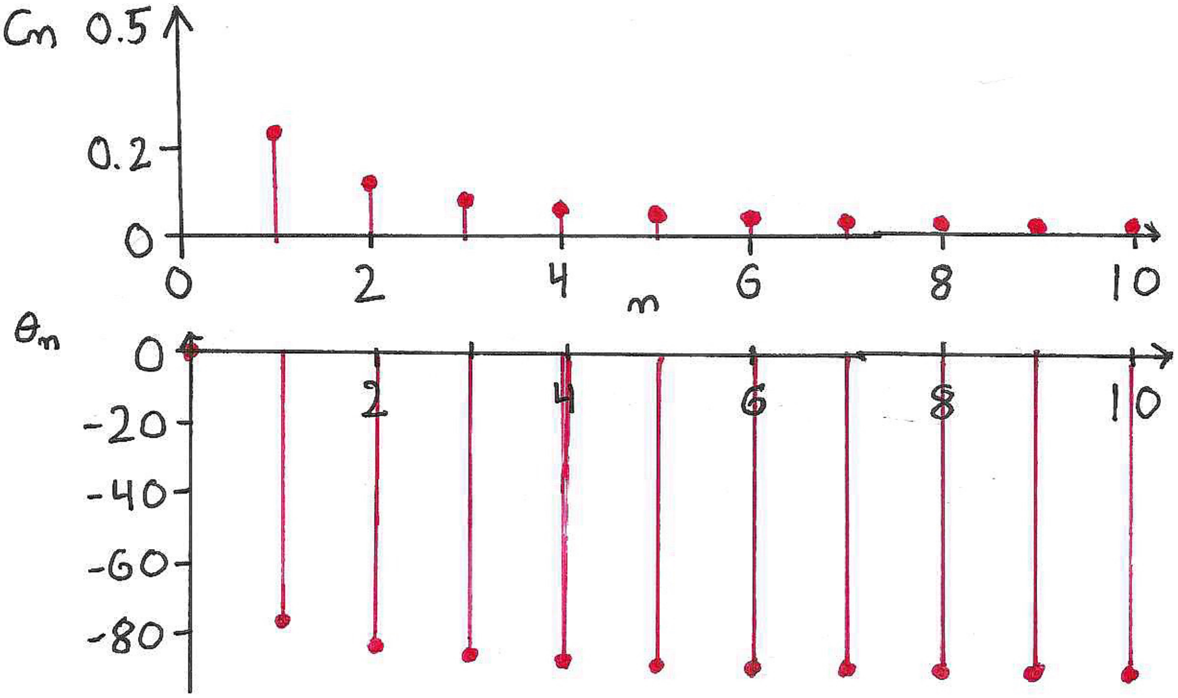
$$a_1 = \int_0^T x(t) \cos(\omega_0 t)\, dt = \int_0^T x(t) \cos\left(\frac{2\pi}{T}\right)t\, dt = 0.059$$

$$b_1 = \int_0^T x(t) \sin(\omega_0 t)\, dt = \int_0^T x(t) \sin\left(\frac{2\pi}{T} t\right) dt = 0.23$$

$$C_1 = \sqrt{0.059^2 + 0.23^2} = 0.244$$

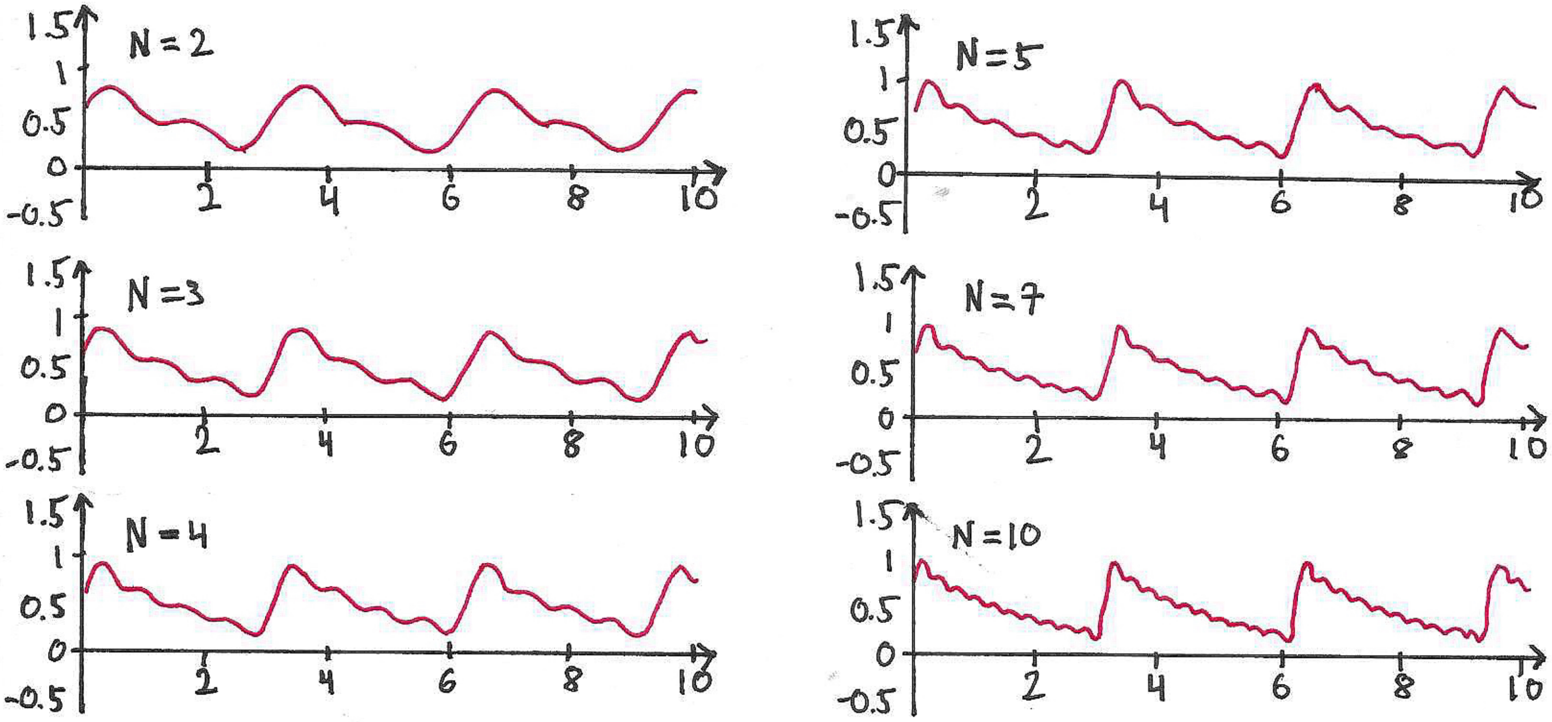
$$\theta_1 = \frac{180^\circ}{\pi} \cdot \tan^{-1}\left(-\frac{0.23}{0.059}\right) = -76^\circ$$

⋮



These plots shows which frequencies are contained in $x(t)$ and how <u>much</u>.

We can <u>synthesize</u> an approximation to $x(t)$ using the coefficients.

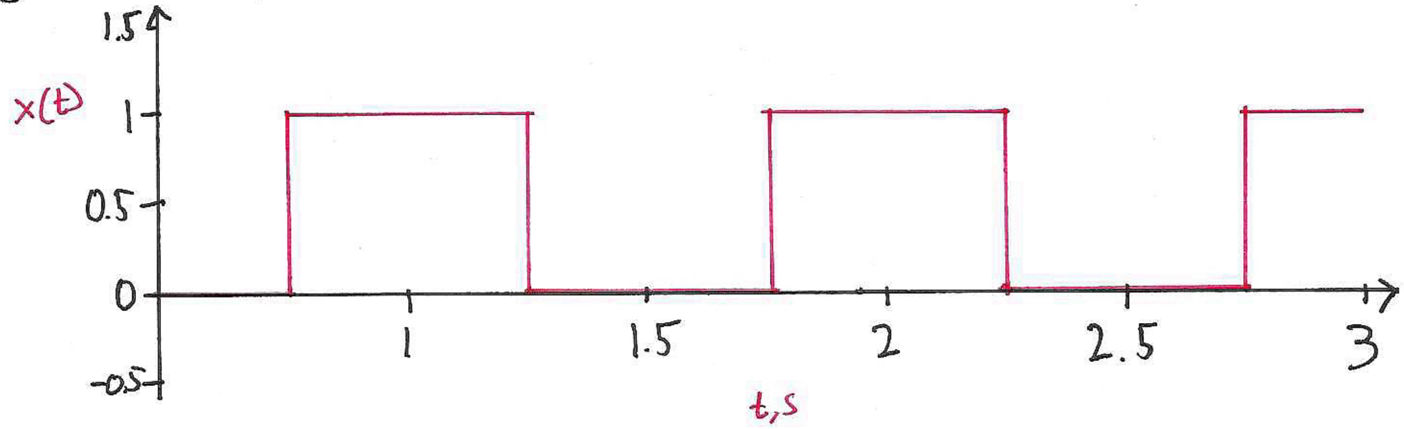


The more harmonics (N) we include the better approximation.

$x(t)$ is discontinuous in $t=\pi$, so the Fourier Series is <u>not</u> uniformly convergent.

Problem 4

Perform complex Fourier expansion on the square pulse signal.



Sol

Periodic signal with period $T = 1\,s$.

Complex Fourier coefficients: $D_n = \frac{1}{T} \cdot \int_0^T x(t) e^{-j\omega_0 t} \, dt$

$x(t)$ is real and even, so we expect real coefficients.

$$D_0 = \frac{1}{T} \int_0^T x(t)\, dt = 0.5$$

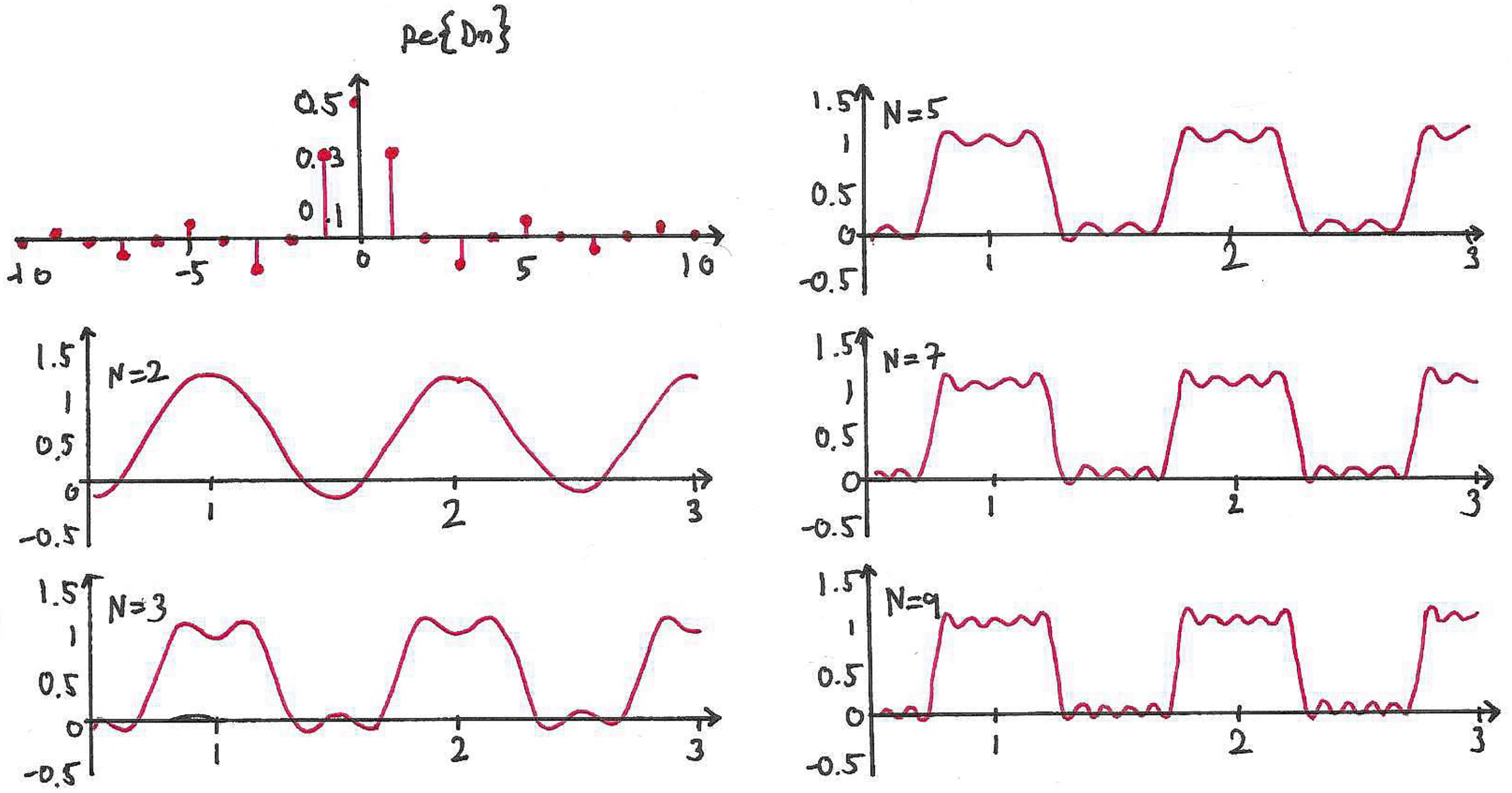
$$D_{-1} = D_1 = \frac{1}{T} \int_0^T x(t) e^{-j\frac{2\pi}{T}t} \, dt = 0.318$$

$$D_{-2} = D_2 = \frac{1}{T} \int x(t) e^{-j2\cdot\frac{2\pi}{T}t} \, dt = 0$$

$$D_{-3} = D_3 = \frac{1}{T} \int x(t) e^{-j3\cdot\frac{2\pi}{T}\cdot t} \, dt = -0.106$$

$$D_{-4} = D_4 = \frac{1}{T} \int x(t) e^{-j4\cdot\frac{2\pi}{T}\cdot t} \, dt = 0$$

The square wave only has odd harmonics.



No imaginary part, so only $Re\{D_n\}$ is plotted.

The approximation gets better by including higher order coefficients.

$D_n = D_{-n}$ must hold $\rightarrow$ complex Fourier coefficients are symmetric around $n=0$.

Fourier series <u>not</u> uniformly convergent.

$$F(1) = \frac{F(\frac{1}{2}^+) + F(\frac{1}{2}^-)}{2} = \frac{1}{2}$$

No imaginary part $\rightarrow$ no phase.