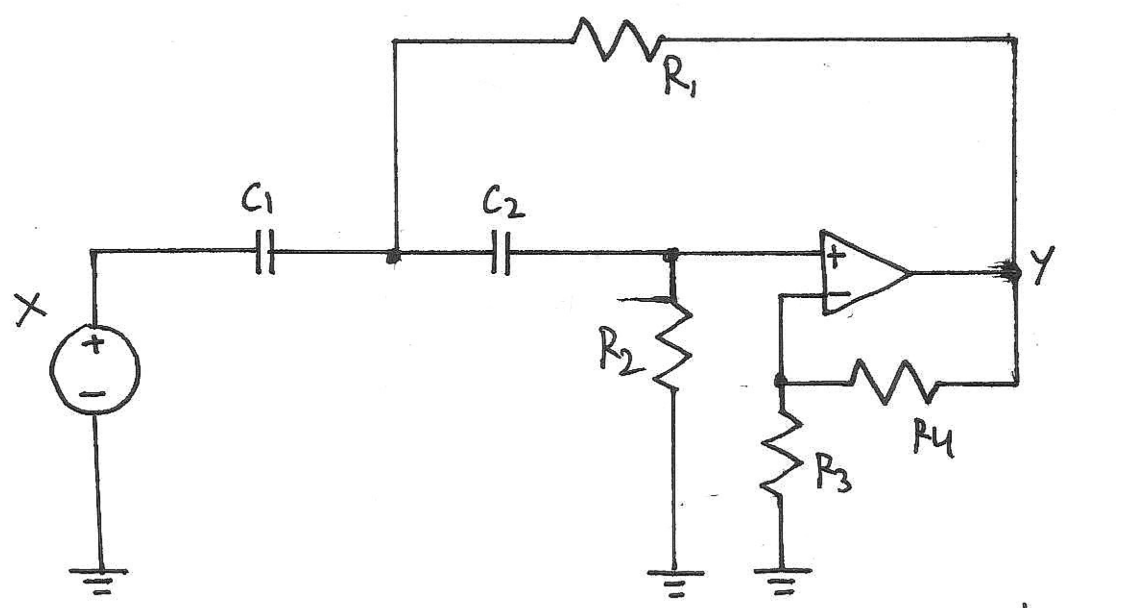
SIGNALS AND SYSTEMS - WEEK 3



Impulse response: $h(t) = 2\delta(t) + 0.6283(-2+0.314t)e^{-0.314t}\,u(t)$

Problem 1

Determine if the system is asymptotically stable, marginally stable or unstable.

sol

Recall the roots of the characteristic polynomial (i.e. the poles of the system) were

$$\lambda = -\frac{\pi}{10} \quad (am = 2)$$

All roots have negative real part $\rightarrow$ Asymptotically stable.

An asymptotically stable is also BIBO-stable.

Problem 2

Plot $|H(j\omega)|$ in decibels.

sol

What is $H(j\omega)$?

For any linear system

$$Q(D)\,y(t) = P(D)\,x(t)$$

if the input is an exponential $x(t) = e^{j\omega t}$ then the output is a scaled any phase shifted exponential $y(t) = H(j\omega)\,e^{j\omega t}$.

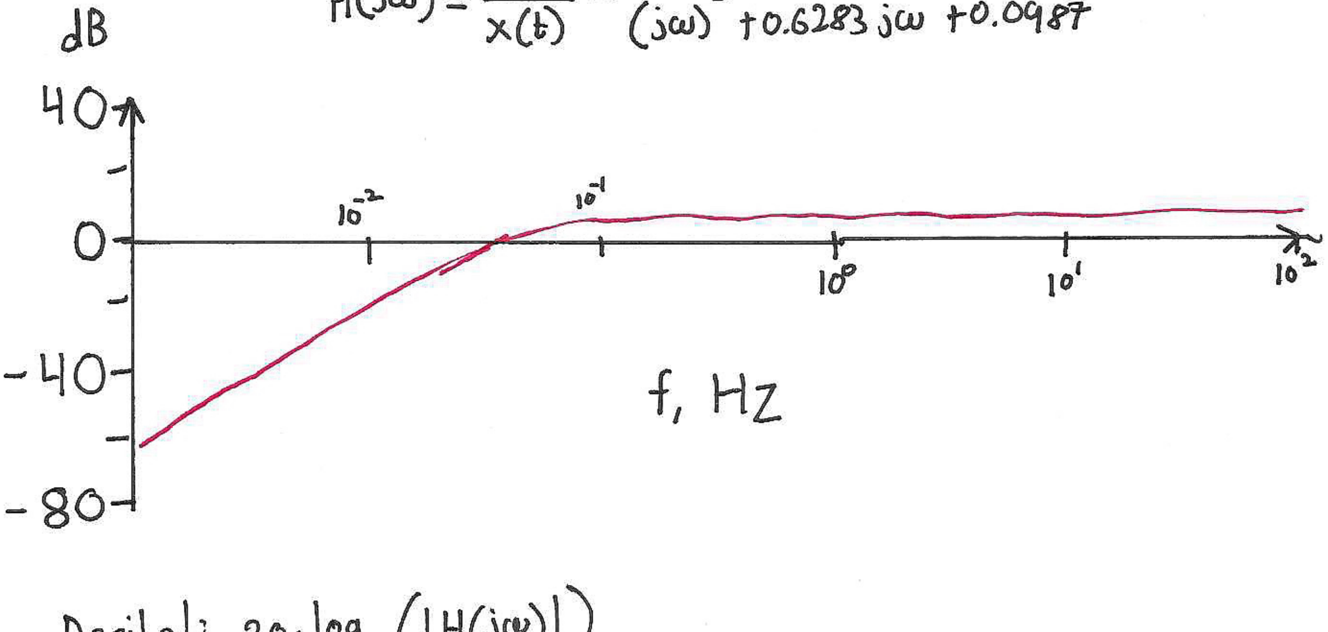
How to find $H(j\omega)$?

Insert $j\omega$ into $Q(D)$ and $P(D)$, and arrive at $\frac{y(t)}{x(t)}$.

$$(D^2 + 0.6283\,D + 0.0987)\,y(t) = 2D^2\,x(t)$$

$$\frac{y(t)}{x(t)} = \frac{2D^2}{D^2 + 0.6283D + 0.0987}$$

$$H(j\omega) = \frac{y(t)}{x(t)} = \frac{2(j\omega)^2}{(j\omega)^2 + 0.6283\,j\omega + 0.0987}$$



Decibel: $20 \cdot \log_{10}(|H(j\omega)|)$

Problem 3

Apply the convolution integral to find the zero-state response to the input $x(t) = 2u(t)$.

sol

$$Y_{zs}(t) = x(t) * h(t)$$

$$Y_{zs}(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Using Maple:

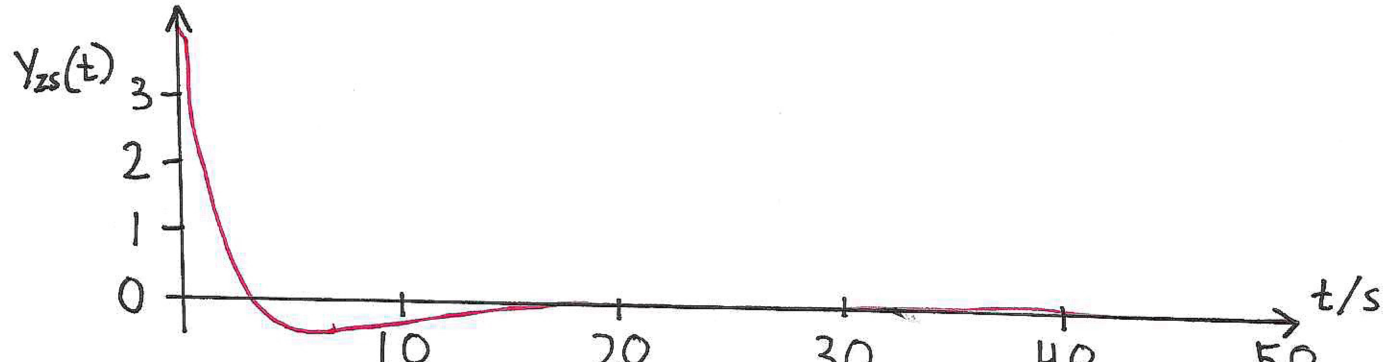
$x := t \rightarrow 2 \cdot \text{Heaviside}(t)$

$h := t \rightarrow \underbrace{2 \cdot \text{Dirac}(t)}_{\text{Impulse}} - 0.6283(2 - 0.314t)\, e^{-0.314t} \cdot \text{Heaviside}(t)$

$$\text{evalf}\left(\int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau\right) = 4 \cdot \text{Heaviside}(t) - 4 - 1.256\, t\, e^{-0.314t} + 4e^{-0.314t}$$

Our system is causal, so only valid for $t \geq 0$.

$$Y_{zs}(t) = 4(1 - 0.314t)\, e^{-0.314t} \cdot U(t)$$



Hints

How to see if filter is low, high, or bandpass?

Answer: Look at differential equation.

$\ddot{y} + a_1\dot{y} + a_0 y = b_0 x \longrightarrow$ Lowpass

$\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = b_1\dot{x}(t) \longrightarrow$ Bandpass

$\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = b_2\ddot{x}(t) \longrightarrow$ Highpass

other way: Look at $H(j\omega)$.

$$H(j\omega) = \frac{b_0}{(j\omega)^2 + a_1 j\omega + a_0} \longrightarrow \text{Lowpass}$$

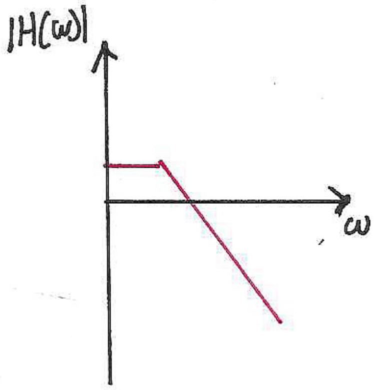
$$H(j\omega) = \frac{b_1 \cdot j\omega}{(j\omega)^2 + a_1 j\omega + a_0} \longrightarrow \text{Bandpass}$$

$$H(j\omega) = \frac{b_2 (j\omega)^2}{(j\omega)^2 + a_1 j\omega + a_0} \longrightarrow \text{Highpass.}$$

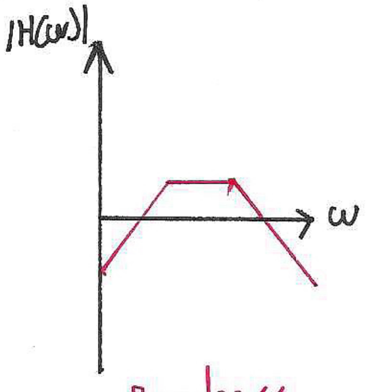
Observation:

The denominator of $H(j\omega)$ always has the same form, but the numerator changes.

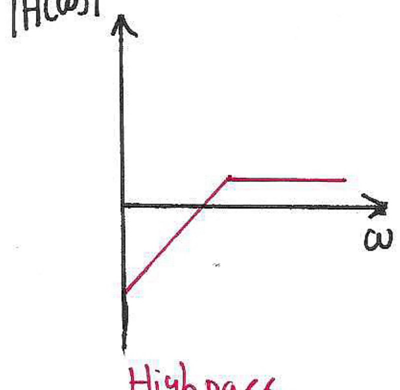
Therefore, the numerator of $H(j\omega)$ determines the filter type.



Lowpass



Bandpass



Highpass.