SIGNALS AND SYSTEMS - WEEK 11

Problem 1

Draw a straight line approximation of the Bode Plot for $G(j\omega) = \dfrac{10^4 (j\omega + 2)}{(j\omega + 10)(j\omega + 100)}$

Sol

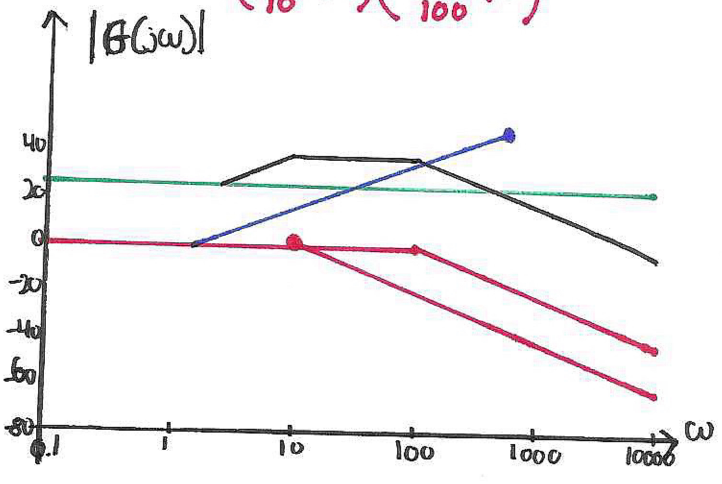
Convert to Bode form, i.e. $\left(\frac{j\omega}{k} + 1\right)$:

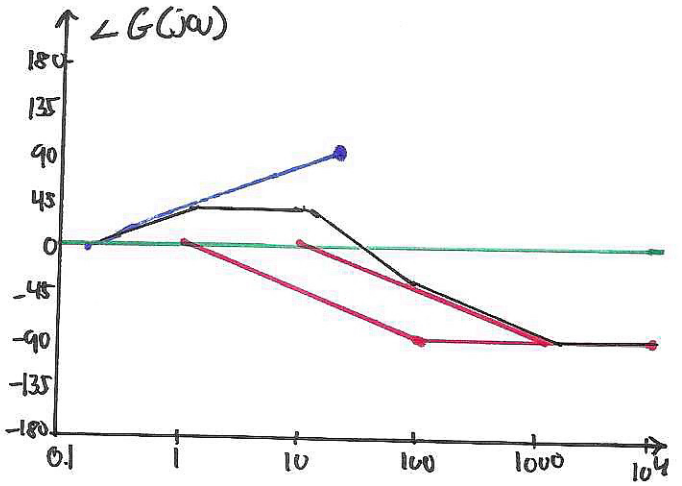
$$G(j\omega) = 10^4 \cdot \frac{2\left(\frac{j\omega}{2} + 1\right)}{10\left(\frac{j\omega}{10} + 1\right)\left(\frac{j\omega}{100} + 1\right) \cdot 100} \iff$$

$$G(j\omega) = \frac{2 \cdot 10^4}{10 \cdot 100} \cdot \frac{\left(\frac{j\omega}{2} + 1\right)}{\left(\frac{j\omega}{10} + 1\right)\left(\frac{j\omega}{100} + 1\right)} \iff$$

$$G(j\omega) = 20 \cdot \frac{\left(\frac{j\omega}{2} + 1\right)}{\left(\frac{j\omega}{10} + 1\right)\left(\frac{j\omega}{100} + 1\right)}$$

(20 = 26 dB)





Remember:

Gain increases/decreases at the zero/pole frequency.

Phase increases/decreases at 1 decade before and stops 1 decade after the zero/pole frequency.

Problem 2

Draw the Bode plot for $H(s) = \dfrac{100s}{s^2 + 20s + 10000}$.

sol

This is an underdamped system ($\zeta < 1$), so we don't factor.

$$H(j\omega) = 100 \cdot \frac{j\omega}{(j\omega)^2 + 20j\omega + 10000} \iff$$

$$H(j\omega) = \frac{100}{100^2} \cdot \frac{j\omega}{\left(\frac{j\omega}{100}\right)^2 + \frac{20}{100} \cdot \frac{j\omega}{100} + 1} \iff$$

$$H(j\omega) = \frac{\frac{j\omega}{100}}{\left(\frac{j\omega}{100}\right)^2 + 2 \cdot 0.1 \frac{j\omega}{100} + 1}$$ (zero in origin, pole in $\omega = 100 \frac{rad}{s}$)

Damping ratio: $\zeta = 0.1$

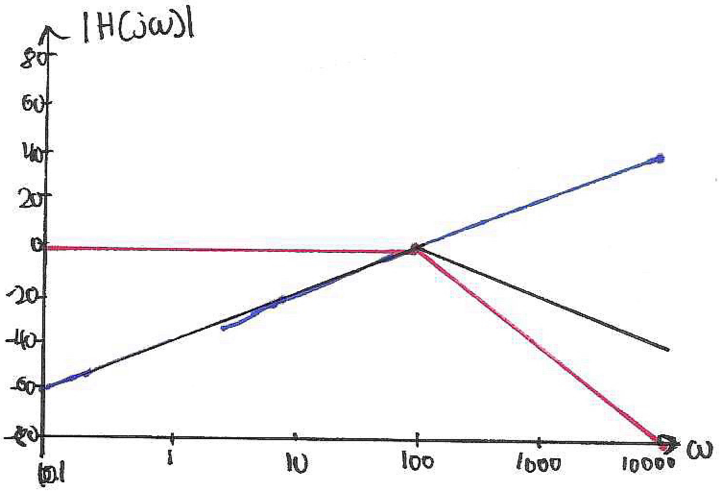
Undamped resonance frequency: $\omega_n = 100 \frac{rad}{s}$.

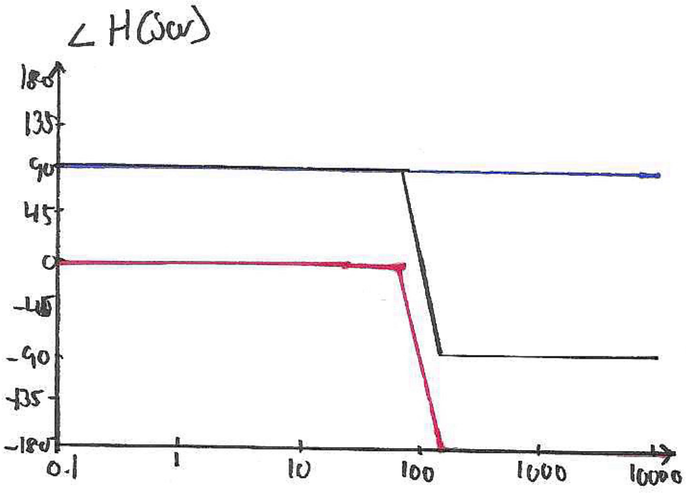
Calculate α: $\alpha = 1.410\zeta - 0.150\zeta^2 = 0.1395$ ($\zeta \leq 0.2$)

$$\omega_{start} = \frac{\omega_n}{10^{\alpha}} = \frac{100}{10^{0.1395}} = 72.5 \frac{rad}{s}$$

$$\omega_{stop} = 10^{\alpha} \cdot \omega_n = 10^{0.1395} \cdot 100 = 137.88 \frac{rad}{s}$$

} Needed for phase plots.





Amplitude approximation is not accurate. Systems with $\zeta < \frac{1}{\sqrt{2}}$ have a peak in $|H(j\omega)|$.

Problem 3

Draw Bode plot for $H(s) = \dfrac{789629}{s^2 + 2827.53s + 394815}$.

sol

This is overdamped, so we factor.

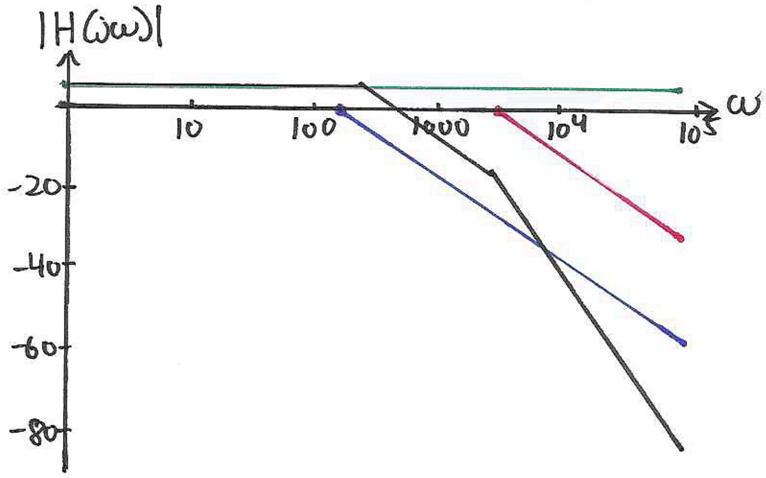
$$H(s) = \frac{789629}{(s+147.3)(s+2680)} \Leftrightarrow$$

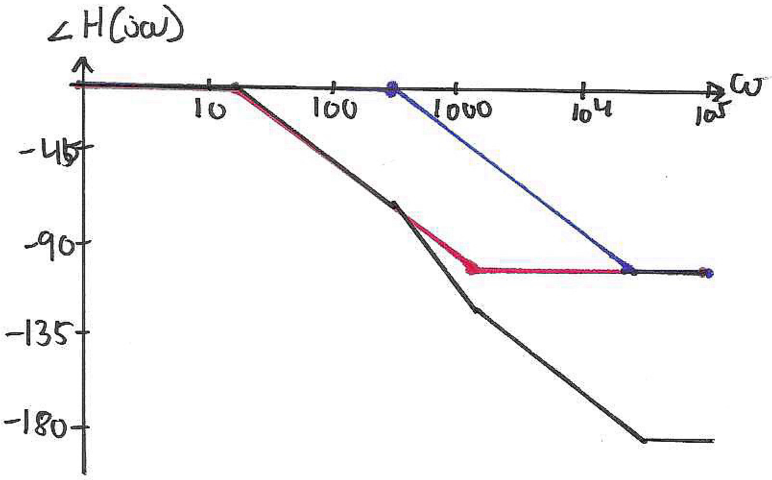
$$H(j\omega) = \frac{789629}{147.3\left(\frac{j\omega}{147.3}+1\right)\left(\frac{j\omega}{2680}+1\right)2680} \Leftrightarrow$$

$$H(j\omega) = \frac{789629}{147.3 \cdot 2680} \cdot \frac{1}{\left(\frac{j\omega}{147.3}+1\right)\left(\frac{j\omega}{2680}+1\right)}$$

$$H(j\omega) = \underbrace{2}_{=6\,dB} \cdot \frac{1}{\left(\frac{j\omega}{147.3}+1\right)\left(\frac{j\omega}{2680}+1\right)}$$

} No zeros
Distinct real poles





- A pole decreases gain by $20 \frac{dB}{dec}$.
- A pole decreases phase by a total of $90°$.

Problem 4

Draw Bode plot of $H(s) = \frac{2s^2}{(s+\frac{\pi}{10})^2}$

Sol

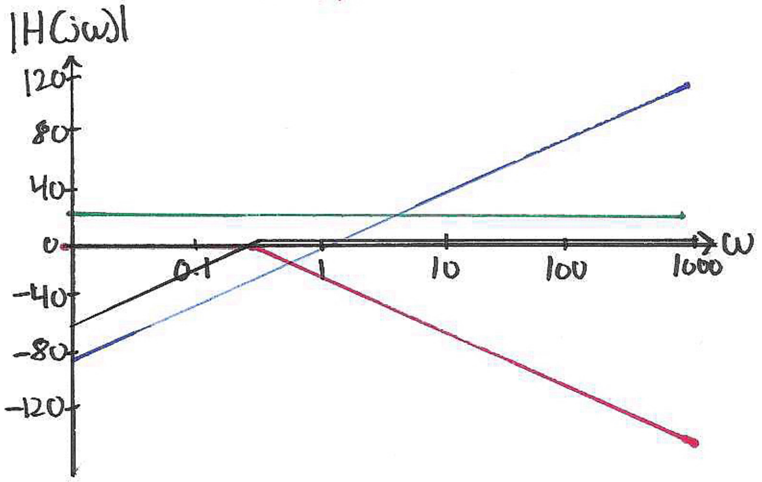
This is critically damped, so we keep the factored form.

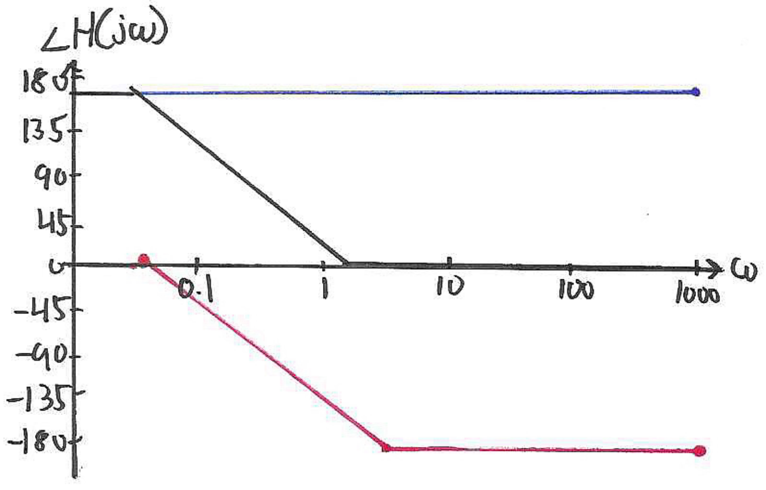
$$H(s) = 2 \cdot \frac{s^2}{\left(\frac{\pi}{10}\right)^2 \left(\frac{s}{\pi/10}+1\right)^2} \iff$$

$$H(j\omega) = \frac{2}{(\pi/10)^2} \cdot \frac{(j\omega)^2}{\left(\frac{j\omega}{\pi/10}+1\right)^2}$$

Double pole at $-\frac{\pi}{10}$ and double zero in origin.

$$H(j\omega) = 2 \cdot \frac{\left(\frac{j\omega}{\pi/10}\right)^2}{\left(\frac{j\omega}{\pi/10}+1\right)^2} = 20.26 \frac{(j\omega)^2}{\left(\frac{j\omega}{\pi/10}+1\right)^2}$$





- Double zero in origin: Phase starts in $+180^0$.
- Double pole in $-\frac{\pi}{10}$: Gain slope after $\omega = \frac{\pi}{10}$ is $2 \cdot (-20) \frac{dB}{dec} = -40 \frac{dB}{dec}$.

Problem 5

Draw Bode plot of $H(s) = \dfrac{628.3s}{s^2 + 31.41s + 98695}$.

sol

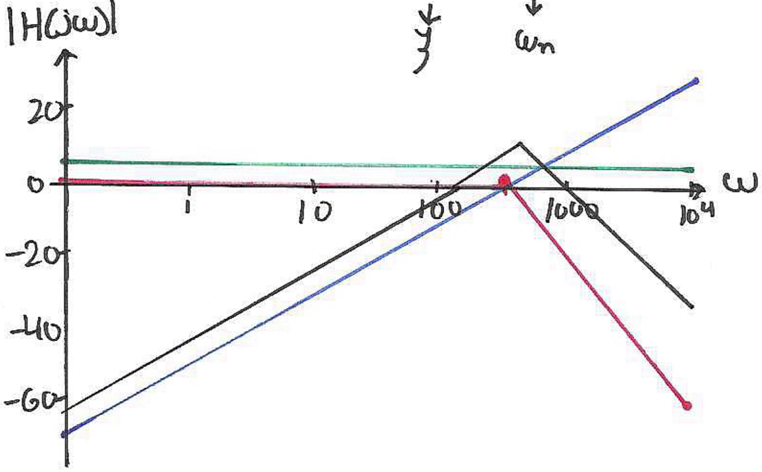
This system is underdamped ($\zeta = 0.05$), so we don't factor.

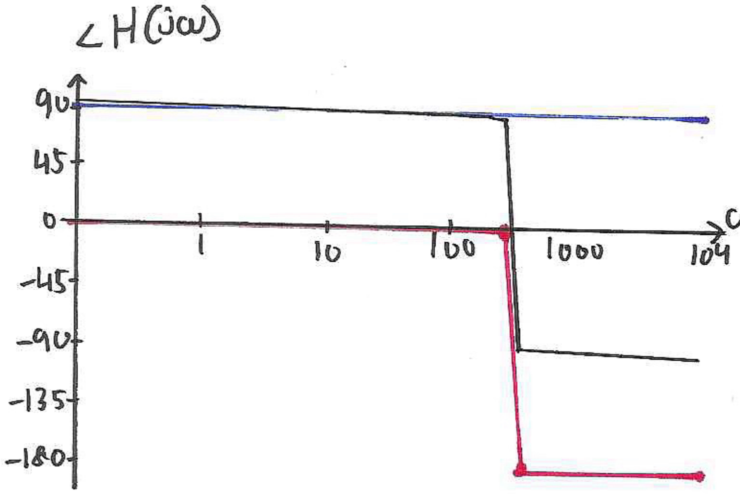
$98695 = 314^2$

$$H(j\omega) = 628.3 \cdot \frac{j\omega}{(j\omega)^2 + 31.41 j\omega + 314^2} \iff$$

$$H(j\omega) = \frac{628.3}{314^2} \cdot \frac{j\omega}{\left(\frac{j\omega}{314}\right)^2 + \frac{31.41}{314} \cdot \frac{j\omega}{314} + 1} \iff$$

$$H(j\omega) = 2 \cdot \frac{\frac{j\omega}{314}}{\left(\frac{j\omega}{314}\right)^2 + \underbrace{\frac{31.41}{314}}_{\zeta} \cdot \underbrace{\frac{j\omega}{314}}_{\omega_n} + 1}$$





For phase plot: $\alpha = 1.410\zeta - 0.150\zeta^2 = 0.0701$

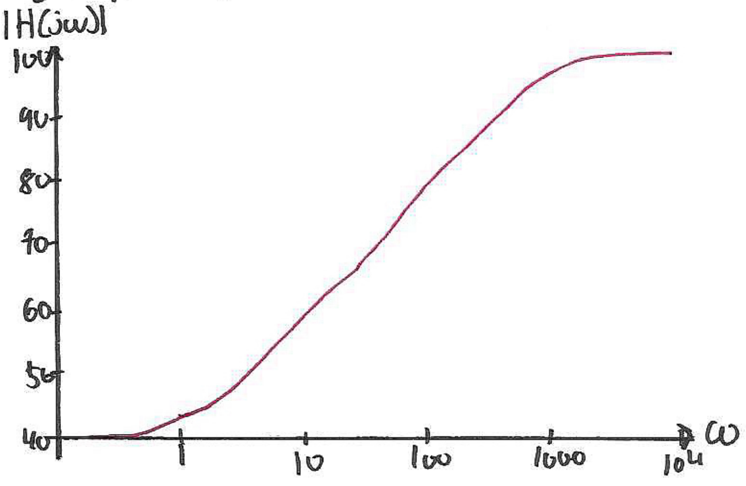
$$\omega_{start} = \frac{\omega_n}{10^\alpha} = \frac{314}{10^{0.0701}} = 267.3 \ \frac{rad}{s}$$

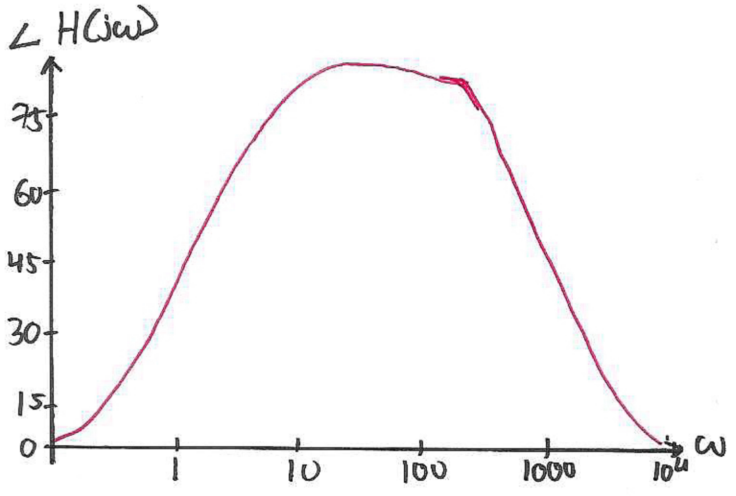
$$\omega_{stop} = \omega_n 10^\alpha = 314 \cdot 10^{0.0701} = 369.2 \ \frac{rad}{s}$$

Not accurate amplitude spectrum/response.

Problem 6

Write the transfer function for the system with these frequency characteristics.





Sol

The amplitude response "cuts" at $\omega = 1$ and $\omega = 1000$.

First cut is a zero (increasing gain).

Second cut is a pole ("decreasing" gain).

A DC gain of 40 dB = 100.

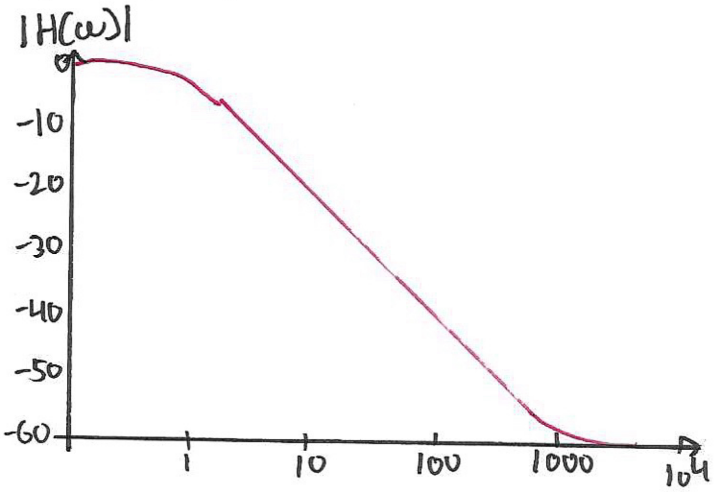
$$H(j\omega) = 100 \cdot \frac{\left(\frac{j\omega}{1} + 1\right)}{\left(\frac{j\omega}{1000} + 1\right)}$$

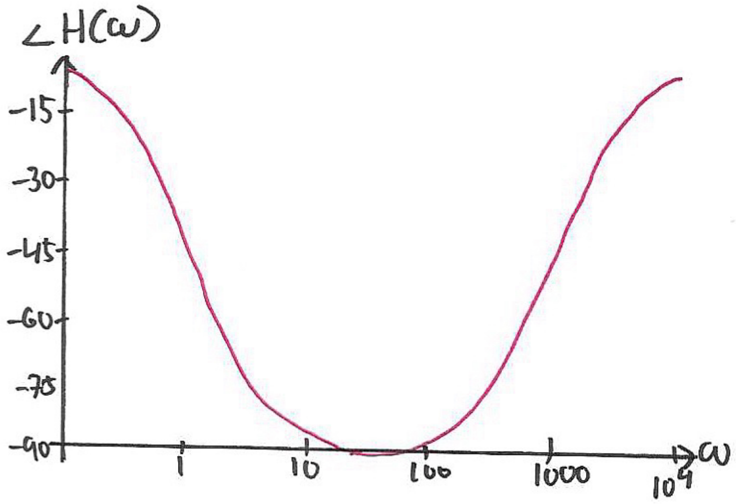
$$H(j\omega) = 100 \cdot \frac{(j\omega + 1)}{\frac{1}{1000}(j\omega + 1000)} = \frac{100}{1/1000} \cdot \frac{j\omega + 1}{j\omega + 1000}$$

$$H(s) = 10^5 \cdot \frac{s+1}{s+1000}$$

Problem 7

From the frequency characteristic, write the transfer function.





Sol

- DC gain $K = 0$ dB $= 1$.
- Gain decreases at $\omega = 1$ (pole), by $20 \frac{dB}{dec}$
- Gain stops decreasing at $\omega = 1000$ (zero).

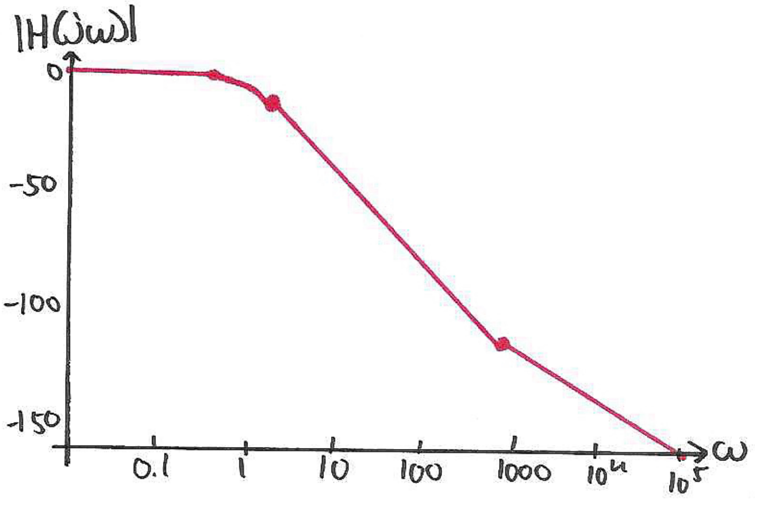
$$H(j\omega) = 1 \cdot \frac{\left(\frac{j\omega}{1000} + 1\right)}{\frac{j\omega}{1} + 1} \Leftrightarrow$$

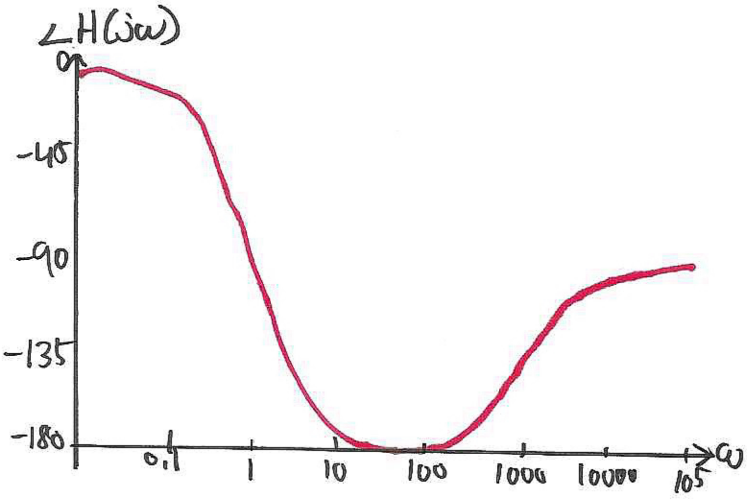
$$H(j\omega) = 1 \cdot \frac{\frac{1}{1000}(j\omega + 1000)}{j\omega + 1}$$

$$H(s) = 10^{-3} \cdot \frac{s + 1000}{s + 1}$$

Problem 8

Find $H(s)$ from the frequency characteristic.





Sol

- DC gain $k = 0\ dB = 1$.
- Gain decreases by $40 \frac{dB}{dec}$ at $\omega = 1$ (double pole)
- Gain decreases by $20 \frac{dB}{dec}$ at $\omega = 1000$ (zero).

$$H(j\omega) = 1 \cdot \frac{\left(\frac{j\omega}{1000} + 1\right)}{\left(\frac{j\omega}{1} + 1\right)^2}$$

$$H(j\omega) = 1 \cdot \frac{\frac{1}{1000} \cdot (j\omega + 1000)}{(j\omega + 1)^2}$$

$$H(s) = 10^{-3} \cdot \frac{s + 1000}{(s+1)^2}$$